Lightest Neutralino in Extensions of the MSSM

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Abstract

We study neutralino sectors in extensions of the MSSM that dynamically generate the μ-term. The extra neutralino states are superpartners of the Higgs singlets and/or additional gauge bosons. The extended models may have distinct lightest neutralino properties which can have important influences on their phenomenology. We consider constraints on the lightest neutralino from LEP, Tevatron, and $(g-2)_\mu$ measurements and the relic density of the dark matter. The lightest neutralino can be extremely light and/or dominated by its singlino component which does not couple directly to SM particles except Higgs doublets.

I. INTRODUCTION

One of the most important challenges to the Standard Model (SM) of particle physics is the cosmological observation that the dominant component of matter in the universe is unexplained. The preferred interpretation of the observational data is that this matter is massive (cold), electrically neutral (dark), and also stable or long-lived. The Supersymmetric extension of the SM provides a well-motivated candidate for cold dark matter. Supersymmetry (SUSY) predicts the superpartners of the neutral gauge bosons and the neutral Higgs bosons with masses of electroweak (EW) scale and couplings of EW strength. The lightest neutralino is the lightest Supersymmetric particle (LSP) over most of the parameter space. Assuming R-parity conservation that prevents rapid proton decay, the lightest neutralino is stable.

The Minimal version of the Supersymmetric SM (MSSM) assumes R-parity conservation and has an extra Higgs doublet and general SUSY breaking soft terms [1]. Extensive studies of the MSSM show that its lightest neutralino has the right ranges of mass and interaction strength to be a good cold dark matter (CDM) candidate [2]. Although the MSSM may be the optimal low energy Supersymmetric model with minimal extension of the fields and symmetry, the model may not fully describe the TeV scale physics. It is important to see if other versions of the Supersymmetric SM can also give acceptable CDM.

A good starting point for alternatives to the MSSM is its theoretical weakness. The MSSM has a fine-tuning problem, the so called μ-problem [3]. The μ-term is the only dimensionful parameter in the SUSY conserving sector of the MSSM, and it is required to have the same scale as the SUSY breaking parameters (EW/TeV-scale soft terms) to give an EW scale Higgs vacuum expectation value (VEV) without fine-tuning. The MSSM by itself does not explain why the μ parameter should be at the EW scale. Considering that the motivation of the supersymmetrization of the SM was to resolve a fine-tuning problem (gauge hierarchy problem), the fine-tuning problem of the MSSM is a serious issue.

We consider various extended MSSM models that resolve this μ-problem and compare their lightest neutralino properties to that of the MSSM. In these beyond-MSSM models, at least one Higgs singlet is commonly present and generates an effective μ parameter when the associated symmetry is broken at the EW or TeV scale. Because of the superpartners of the Higgs singlets or the extra gauge boson, the neutralino sector in these models may be significantly different from that of the MSSM. We investigate the mass and the coupling of the lightest neutralino of each model allowed by the model parameters and the experimental data.

In Section II, we describe the models and their neutralino sectors. In Section III, we analyze the lightest neutralinos allowed by the model parameters and direct experimental constraints. In Section IV, we discuss indirect observational constraints on these lightest neutralinos. In Section V, we discuss our numerical results, and then summarize our results in Section VI.

II. MODELS

The extended MSSM models that we consider are the Next-to-Minimal Supersymmetric SM (NMSSM) [4], the Minimal Non-minimal Supersymmetric SM (MNSSM) a.k.a. the nearly Minimal Supersymmetric SM (nMSSM) [5], the $U(1)'$-extended Minimal Supersymmetric SM (UMSSM) [6], and the $U(1)'$-extended Supersymmetric SM with a secluded $U(1)'$-

TABLE I: Higgses and Neutralinos of the MSSM and its extensions

Model	Symmetry	Higgses (CP even, CP odd, charged)	Neutralinos
MSSM	–	$H_1^0, H_2^0, A^0, H^\pm$	$\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0$
NMSSM	$\mathbb{Z}_3$	$+ H_3^0, A_2^0$	$+ \tilde{S}$
nMSSM	$\mathbb{Z}_5^R, \mathbb{Z}_7^R$	$+ H_3^0, A_2^0$	$+ \tilde{S}$
UMSSM	$U(1)'$	$+ H_3^0$	$+ \tilde{S}, \tilde{Z}'$
S-model	$U(1)'$	$+ H_3^0, H_4^0, H_5^0, H_6^0, A_2^0, A_3^0, A_4^0$	$+ \tilde{S}, \tilde{Z}', \tilde{S}_1, \tilde{S}_2, \tilde{S}_3$

breaking sector (S-model) [7]. All of these extended models prevent the μ-term ($\mu H_1 H_2$) and allow an effective μ-term ($S H_1 H_2$) through a VEV $\langle S \rangle$ of a Higgs singlet associated with a new symmetry. The NMSSM and the nMSSM adopt discrete symmetries while the UMSSM and the S-model use an Abelian gauge symmetry spontaneously broken by the Higgs singlet.

A. Superpotentials

The superpotential of Higgses (both isospin doublets and singlets) for each model is given below[1].

$$W_{\text{MSSM}} = \mu H_1 H_2 \tag{1}$$

$$W_{\text{NMSSM}} = h_s S H_1 H_2 + \frac{\kappa}{3} S^3 \qquad W_{\text{nMSSM}} = h_s S H_1 H_2 + \alpha S \tag{2}$$

$$W_{\text{UMSSM}} = h_s S H_1 H_2 \qquad W_{\text{S-model}} = h_s S H_1 H_2 + \lambda_s S_1 S_2 S_3 \tag{3}$$

The other parts of the superpotentials are the Yukawa terms of the MSSM and possible extra terms related to the exotic chiral fields in the $U(1)'$ models needed to cancel anomalies. The exotic chiral terms are model-dependent and we do not specify them here, assuming that the masses of exotic fields are heavy enough to give insignificant effects to the EW scale phenomenology that we are interested in. Specific examples of models with exotic fields can be found in Ref. [6, 7].

The Higgses and neutralinos of each model are listed in Table I. In the $U(1)'$-extended model, the addition of one Higgs singlet does not give an additional CP odd Higgs since a goldstone boson is absorbed to be the longitudinal mode of the massive $U(1)'$ gauge boson, Z'.

We use the common notation of h_s for the coefficient of the $S H_1 H_2$ term in each model for easy comparison. In every model the dynamically generated effective μ parameter is given by

$$\mu_{\text{eff}} = h_s \langle S \rangle \tag{4}$$

and therefore the VEV of the Higgs singlet or the symmetry breaking scale needs to be at the EW/TeV scale.

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[1] The term αS in the nMSSM is a loop-generated tadpole term that breaks the discrete symmetry; see Ref. [5].

B. Neutralino mass matrices

The MSSM has 4 neutralinos ($\tilde{B}$, $\tilde{W}_3$, $\tilde{H}_1^0$, $\tilde{H}_2^0$) while the NMSSM has 5 neutralinos (MSSM components + $\tilde{S}$). The neutralino mass matrix of the NMSSM in the $\{\tilde{B}$, $\tilde{W}_3$, $\tilde{H}_1^0$, $\tilde{H}_2^0$, $\tilde{S}\}$ basis is given by

$$
M_{\chi^0} = \begin{pmatrix}
M_1 & 0 & -g_1 v_1/2 & g_1 v_2/2 & 0 \\
0 & M_2 & g_2 v_1/2 & -g_2 v_2/2 & 0 \\
-g_1 v_1/2 & g_2 v_1/2 & 0 & -\mu_{\text{eff}} & -\mu_{\text{eff}} v_2/s \\
g_1 v_2/2 & -g_2 v_2/2 & -\mu_{\text{eff}} & 0 & -\mu_{\text{eff}} v_1/s \\
0 & 0 & -\mu_{\text{eff}} v_2/s & -\mu_{\text{eff}} v_1/s & 2\kappa s/\sqrt{2}
\end{pmatrix} .
\tag{5}
$$

The first 4×4 submatrix corresponds to the MSSM limit, which can be obtained from Eq. (5) by taking $s \gg \mathcal{O}(\text{EW})$. Depending on the value of the parameter κ, the 5th component may be very heavy and decoupled from the other EW scale components or almost massless. The NMSSM assumes a discrete symmetry $\mathbb{Z}_3$ to avoid the μ-term, but allows S^3 term in the superpotential. The VEVs of the Higgses are defined as $\langle H_i^0 \rangle \equiv \frac{v_i}{\sqrt{2}}$ with $\sqrt{v_1^2 + v_2^2} \equiv v \simeq 246$ GeV and $\langle S \rangle \equiv \frac{s}{\sqrt{2}}$. The gauge couplings are $g_1 = e/\cos\theta_W$ and $g_2 = e/\sin\theta_W$. The NMSSM is one of the simplest extensions of the MSSM, but the $\mathbb{Z}_3$ symmetry predicts domain walls which are not observed [8].

The nMSSM [5] was devised to avoid the domain wall problem while maintaining a discrete symmetry. The nMSSM has the same 5 neutralinos and the same mass matrix as Eq. (5) except that the κ term in the $(5,5)$ entry vanishes since it is not allowed by the discrete symmetry.

The UMSSM uses an Abelian gauge symmetry instead of a discrete symmetry and thus is free from the domain wall problem. With the superpartner of the $U(1)'$ gauge boson (Z'), the UMSSM has 6 neutralinos (NMSSM or nMSSM components + $\tilde{Z}'$) and its mass matrix in the basis of $\{\tilde{B}$, $\tilde{W}_3$, $\tilde{H}_1^0$, $\tilde{H}_2^0$, $\tilde{S}$, $\tilde{Z}'\}$ is given by

$$
M_{\chi^0} = \begin{pmatrix}
M_1 & 0 & -g_1 v_1/2 & g_1 v_2/2 & 0 & 0 \\
0 & M_2 & g_2 v_1/2 & -g_2 v_2/2 & 0 & 0 \\
-g_1 v_1/2 & g_2 v_1/2 & 0 & -\mu_{\text{eff}} & -\mu_{\text{eff}} v_2/s & g_{Z'} Q'_{H_1} v_1 \\
g_1 v_2/2 & -g_2 v_2/2 & -\mu_{\text{eff}} & 0 & -\mu_{\text{eff}} v_1/s & g_{Z'} Q'_{H_2} v_2 \\
0 & 0 & -\mu_{\text{eff}} v_2/s & -\mu_{\text{eff}} v_1/s & 0 & g_{Z'} Q'_S s \\
0 & 0 & g_{Z'} Q'_{H_1} v_1 & g_{Z'} Q'_{H_2} v_2 & g_{Z'} Q'_S s & M_{1'}
\end{pmatrix} .
\tag{6}
$$

The first 5×5 submatrix corresponds to the nMSSM limit (or NMSSM limit in the special case $\kappa = 0$) that can be obtained from Eq. (6) by taking $M_{1'} \gg \mathcal{O}(\text{EW})$. In this limit the mass of the Z'-ino becomes very large and this component decouples from the others. Here $g_{Z'}$ is the $U(1)'$ gauge coupling constant and Q' is the $U(1)'$ charge. The $U(1)'$ charges should satisfy

$$
Q'_{H_1} + Q'_{H_2} \neq 0 \qquad Q'_{H_1} + Q'_{H_2} + Q'_S = 0
\tag{7}
$$

in order to replace the μ-term with the effective μ-term dynamically generated by the Higgs singlet S. For the numerical analysis in this paper we use the η-model charge assignments and the Grand Unification Theory (GUT) motivated gauge coupling, $g_{Z'}$:

$$
Q'_{H_1} = \frac{1}{2\sqrt{15}} \qquad Q'_{H_2} = \frac{4}{2\sqrt{15}} \qquad g_{Z'} = \sqrt{\frac{5}{3}} g_1
\tag{8}
$$

The S-model was introduced to resolve tension between the EW scale μ_{eff} and the heavy Z' (up to multi-TeV scale). It is basically the extension of the UMSSM with 3 additional Higgs singlets to provide additional contributions to the Z' mass while keeping $\mu_{\text{eff}} = h_s \langle S \rangle$ at the EW scale. The S-model has 9 neutralinos (UMSSM components + $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$), and its mass matrix has 3 more columns/rows added to Eq. (6). (See Ref. [7] for the S-model and its full 9×9 neutralino mass matrix.) Its first 6×6 submatrix corresponds to the UMSSM limit, which can be realized by taking $s_{1,2,3} \gg \mathcal{O}(\text{EW})$ with λ_s comparable to gauge couplings. However, the most realistic case is for small λ_s and large $s_{1,2,3}$ [7], in which case four of the neutralinos, consisting almost entirely of $\tilde{Z}'$, $\tilde{S}_1$, $\tilde{S}_2$, and $\tilde{S}_3$, essentially decouple from the others. Since the full 9×9 matrix has a number of free parameters, we consider only this decoupling limit when we discuss the light neutralinos, where there are 5 neutralinos with masses and compositions the same as the nMSSM[2].

C. Interesting limits for the neutralino masses

In this section, we analyze the neutralino mass matrix more closely, especially for the nMSSM and UMSSM limits. The diagonalization of the neutralino mass matrix is accomplished via a unitary matrix N as

$$N^T M_{\chi^0} N = \text{Diag}(M_{\chi_1^0}, M_{\chi_2^0}, \cdots). \tag{9}$$

The singlino ($\tilde{S}$) composition of the lightest neutralino (χ_1^0) is $|N_{15}|^2$.

The nMSSM allows the possibility of a very light or massless neutralino which is mainly singlino. This is apparent from (5) in the limit of very large s (and $\kappa = 0$), for which one obtains a massless singlino. However, the tendency for a light singlino-dominated neutralino persists even for smaller s. In fact, it is possible to have an exactly massless eigenvalue, which occurs for

$$\text{Det}(M_{\chi^0}) = \left(\frac{\mu v}{s}\right)^2 \left(M_Z^2(M_1 \cos^2\theta_W + M_2 \sin^2\theta_W) - \mu M_1 M_2 \sin 2\beta\right) = 0, \tag{10}$$

which leads to

$$M_Z^2 \approx 0.8 \mu M_2 \sin 2\beta, \tag{11}$$

where we have assumed the gaugino mass unification condition $M_1 \simeq 0.5 M_2$. Eq. (11) can easily be satisfied with EW/TeV scale values of μ and M_2. It is interesting that this can occur only for $\mu M_2 > 0$, which is favored by the deviation of the muon anomalous magnetic moment data from the SM expectation [10] [11], and also by $b \to s\gamma$ data in most cases [12]. In practice, parameters satisfying condition (11) are often excluded by the chargino mass and Z width constraints discussed in Section III. Nevertheless, there are allowed points lying nearby in which the combination of moderate s and the smallness of $M_Z^2 - 0.8 \mu M_2 \sin 2\beta$

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[2] The four decoupled neutralinos typically consist of one heavy pair involving the $\tilde{Z}'$ and one linear combination of $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$, as well as two more states associated with the orthogonal combinations of $\tilde{S}_{1,2,3}$ [7, 9]. The latter can be light, or even be the lightest neutralino in limiting cases. We do not consider that possibility here.

lead to a very light singlino-dominated neutralino. For large enough s the neutralino can be massless. We will further discuss the massless neutralino scenario later.

Now we consider interesting limits of the UMSSM.

(i) In the case that only $M_{1'}$, the diagonal element of the 6th row/column of Eq. (6), is very large compared to other mass parameters, the $\tilde{Z}'$ will be very heavy and decoupled, leaving a mass matrix similar to that of the nMSSM.

(ii) When only s is very large compared to other mass parameters (with μ_{eff} at the EW scale, which requires $h_s \sim 0$), the elements of the 5th row/column in Eq. (6) are small except for the 6th component. Then the effective mass matrix for the 5th and the 6th components is

$$M_{\chi^0[5,6]} = \begin{pmatrix} 0 & g_{Z'}Q'_S s \\ g_{Z'}Q'_S s & 0 \end{pmatrix}, \tag{12}$$

which gives almost degenerate physical masses of $g_{Z'}|Q'_S|s$. These two heavy states approximately decouple from the other four neutralinos, which are MSSM-like.

(iii) When both $M_{1'}$ and s are large compared to other mass parameters and μ_{eff} is at the EW scale (i.e., h_s is very small), the effective mass matrix for the 5th and 6th components is

$$M_{\chi^0[5,6]} = \begin{pmatrix} 0 & g_{Z'}Q'_S s \\ g_{Z'}Q'_S s & M_{1'} \end{pmatrix} \tag{13}$$

which has eigenvalues

$$\frac{1 - \sqrt{1 + 4\rho^2}}{2} M_{1'} \quad \text{and} \quad \frac{1 + \sqrt{1 + 4\rho^2}}{2} M_{1'} \tag{14}$$

where $\rho \equiv |g_{Z'}Q'_S s/M_{1'}|$. The corresponding neutralino masses in various limits are

$$0, \quad |M_{1'}| \quad (\rho \ll 1) \tag{15}$$
$$\rho|M_{1'}|, \quad \rho|M_{1'}| \quad (\rho \gg 1) \tag{16}$$
$$0.62|M_{1'}|, \quad 1.62|M_{1'}| \quad (\rho = 1) \tag{17}$$

The $\rho \ll 1$ case corresponds to taking the UMSSM to the nMSSM limit ($M_{1'} \gg \mathcal{O}(\text{EW})$) and then to the MSSM limit ($s \gg \mathcal{O}(\text{EW})$) consecutively. Then a very light neutralino state along with a very heavy neutralino state occurs. Besides these lightest (dominated by singlino) and heaviest (dominated by Z'-ino) states, the other neutralinos are basically the same as the MSSM neutralinos. The $\rho \gg 1$ case is similar to the Eq. (12) case. For the range of $\rho = 10 \sim 0.1$, the mass ratio of the lighter state to the heavier one is $1 \sim 100$.

(iv) The case of gaugino mass unification $M_{1'} = M_1 = \frac{5}{3}\frac{g_1^2}{g_2^2}M_2 \simeq 0.5M_2$ will be considered in the next section.

III. DIRECT CONSTRAINTS ON THE LIGHTEST NEUTRALINOS

Here we discuss the allowed mass range of the lightest neutralino after incorporating direct constraints from the LEP experiments and the model parameter structure.

A. LEP constraints on the light chargino mass and the Z width

All the models considered have only one charged gaugino ($\tilde{W}^\pm$) and one charged Higgsino ($\tilde{H}^\pm$) and thus have a common chargino mass matrix which is the same as in the MSSM,

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & \mu_{\text{eff}} \end{pmatrix}. \tag{18}$$

The LEP2 data requires the light chargino mass to be $M_{\chi_1^\pm} > 104$ GeV [13] which gives lower bounds on M_2 and μ_{eff} for a fixed $\tan\beta$.

For a lightest neutralino with $M_{\chi_1^0} \leq M_Z/2$, the LEP constraint $\Gamma_Z^{\text{exp}} - \Gamma_Z^{\text{SM}} = (-2.0 \pm 2.6)$ MeV [14] must be satisfied. Since the Z boson does not couple directly to the singlino or Z'-ino, the Z-χ_1^0-χ_1^0 coupling for every model is the same as that of the MSSM. The expression for the partial width for Z decay to a pair of the lightest neutralinos is

$$\Gamma_{Z \to \chi_1^0 \chi_1^0} = \frac{g_1^2 + g_2^2}{4\pi} \frac{(|N_{13}|^2 - |N_{14}|^2)^2}{24M_Z^2} \left(M_Z^2 - (2M_{\chi_1^0})^2 \right)^{3/2} \Theta(M_Z - 2M_{\chi_1^0}). \tag{19}$$

For a lightest neutralino of very small mass ($M_{\chi_1^0} \ll M_Z/2$) to be allowed[3], its two doublet Higgsino compositions should be either almost identical ($|N_{13}|^2 \approx |N_{14}|^2$) or negligible ($|N_{13}|^2, |N_{14}|^2 \ll 1$) by gaugino or singlino dominance.

B. Lightest neutralino mass range allowed by direct constraints

We evaluate the bounds on the lightest neutralino mass ($M_{\chi_1^0}$) and the singlino component of χ_1^0 in the MSSM, the NMSSM, the nMSSM, and the UMSSM with gaugino mass unification. The nMSSM also represents the NMSSM in the $\kappa \to 0$ limit, the UMSSM in the $M_{1'} \gg M_1$ limit, and the decoupling limit of the S-model.

We require that the tree-level masses satisfy the direct LEP limits of $M_{\chi_1^\pm} > 104$ GeV, and $\Gamma_{Z \to \chi_1^0 \chi_1^0} < 2.3$ MeV (95% C.L.). Bounds from naturalness and perturbativity constraints [7, 15, 16] are also imposed on the couplings of $0.1 \leq h_s \leq 0.75$ and $\sqrt{h_s^2 + \kappa^2} \leq 0.75$ (for the NMSSM)[4]. The LEP2 Higgs mass bound of $m_h > 114$ GeV does not apply directly to the extended MSSM models where the physical Higgses exist as mixtures of doublets and singlets [17].

We choose a phase convention in which μ and the VEVs are real and positive. κ and the gaugino masses can in principle be complex, but we restrict our considerations to real values. We mainly consider the favored case of positive gaugino masses, but comment on the effects of negative masses. We scan M_2, $\mu = 50 \sim 500$ GeV with a step size of 1

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[3] There are some points involving two or more very light neutralinos, for which the lightest neutralino satisfies the Z width constraint but the others do not. This generally occurs for small M_2 and μ values which are already excluded by the chargino mass constraint.

[4] An exact h_s bound (and its source) may be a little different depending on models, but we assume a common bound for easy comparison. A lower bound on the NMSSM $|\kappa|$ is fuzzy except that $\kappa \neq 0$ should be satisfied to avoid an unacceptable Peccei-Quinn symmetry. We set $|\kappa| \geq 0.1$ as a lower bound; results for a smaller $|\kappa|$ can be described by an interpolation of the nMSSM result ($\kappa \to 0$ limit).

TABLE II: The lower (upper) bounds on the lightest neutralino mass ($M_{\chi_1^0}$), the singlino composition ($|N_{15}|^2$) and the parameter values at the bounds in various Supersymmetric models for several fixed $\tan\beta$ values. All mass units are in GeV, except for $\Gamma_{Z\to\chi_1^0\chi_1^0}$ in MeV. For the UMSSM, the η-model charge assignment is assumed. We consider the range $0.1 \leq h_s \leq 0.75$ and only cases where the direct experimental constraints $M_{\chi_1^\pm} > 104$ GeV and $\Gamma_{Z\to\chi_1^0\chi_1^0} < 2.3$ MeV (95% C.L.) are satisfied. The scans are made for M_2, $\mu = 50 \sim 500$ GeV and $s = 50 \sim 2000$ GeV. For the NMSSM, the parameter range $|\kappa| = 0.1 \sim 0.75$ is scanned, and the $\kappa > 0$ and $\kappa < 0$ cases are listed separately.

$\tan\beta$	Model	$M_{\chi_1^0}$	$|N_{15}|^2$	μ	M_2	s	κ	h_s	$M_{\chi_1^\pm}$	$\Gamma_{Z\to\chi_1^0\chi_1^0}$
1	MSSM	55(242)		481(500)	121(500)				104(420)	– (–)
	NMSSM	55(242)	0.00(0.00)	481(500)	121(500)	1850(1605)	0.15(0.30)	0.37(0.44)	104(420)	– (–)
		0(242)	0.78(0.00)	242(500)	265(500)	470(1995)	−0.10(−0.40)	0.73(0.35)	173(420)	0.00(–)
	nMSSM	2(83)	0.99(0.71)	142(121)	486(500)	2000(230)		0.10(0.74)	124(105)	0.00(–)
	UMSSM	0(242)	0.60(0.00)	171(500)	444(500)	365(2000)		0.66(0.35)	149(420)	0.55(–)
2	MSSM	55(243)		486(500)	118(500)				104(425)	– (–)
	NMSSM	48(243)	0.23(0.00)	168(500)	190(500)	320(2000)	0.10(0.65)	0.74(0.35)	104(425)	– (–)
		17(243)	0.78(0.00)	255(500)	500(500)	495(1030)	−0.10(−0.20)	0.73(0.69)	234(425)	2.29(–)
	nMSSM	1(50)	0.98(0.58)	142(151)	248(500)	2000(285)		0.10(0.75)	104(136)	0.03(–)
	UMSSM	26(243)	0.61(0.00)	228(500)	487(500)	430(1200)		0.75(0.59)	208(425)	2.29(–)
10	MSSM	54(247)		468(500)	110(500)				104(441)	– (–)
	NMSSM	21(247)	0.43(0.00)	202(500)	133(500)	405(2000)	0.10(0.65)	0.71(0.35)	104(441)	2.29(–)
		37(247)	0.71(0.00)	216(500)	476(500)	415(980)	−0.10(−0.20)	0.74(0.72)	206(441)	2.30(–)
	nMSSM	0(6)	0.98(0.87)	161(355)	321(500)	2000(670)		0.11(0.75)	145(335)	0.04(2.26)
	UMSSM	39(247)	0.55(0.00)	173(500)	499(500)	330(1180)		0.74(0.60)	165(441)	2.30(–)
50	MSSM	53(248)		451(500)	108(500)				104(445)	– (–)
	NMSSM	16(248)	0.48(0.00)	181(500)	133(500)	405(2000)	0.10(0.65)	0.63(0.35)	104(445)	2.29(–)
		39(248)	0.66(0.00)	191(500)	475(500)	365(980)	−0.10(−0.20)	0.74(0.72)	184(445)	2.30(–)
	nMSSM	0(10)	0.98(0.76)	500(246)	500(118)	2000(590)		0.35(0.59)	445(104)	0.04(2.29)
	UMSSM	41(248)	0.50(0.00)	147(500)	500(500)	280(1170)		0.74(0.60)	143(445)	2.30(–)

GeV[5], $s = 50 \sim 2000$ GeV with a step size of 5 GeV, $\tan\beta = 0.5$, 1, 1.5, 2, 10, 50, and $|\kappa| = 0.1 \sim 0.75$ with a step size of 0.05 (for the NMSSM). The gaugino mass unification relation $M_{1'} = M_1 = \frac{5}{3}\frac{g_1^2}{g_2^2}M_2 \simeq 0.5M_2$ is assumed. The Higgs singlet VEV s cannot be too small in the UMSSM because it is responsible for the mass of the Z', but there are some possible ways to get around this, as we will discuss in Section IV B.

The bounds that we obtain on the the lightest neutralino mass are

$$53 \text{ GeV} \leq M_{\chi_1^0} \leq 248 \text{ GeV} \quad \text{[MSSM]} \tag{20}$$

$$0 \text{ GeV} \leq M_{\chi_1^0} \leq 248 \text{ GeV} \quad \text{[NMSSM]} \tag{21}$$

$$0 \text{ GeV} \leq M_{\chi_1^0} \leq 83 \text{ GeV} \quad \text{[nMSSM, S-model (decoupling limit)]} \tag{22}$$

$$0 \text{ GeV} \leq M_{\chi_1^0} \leq 248 \text{ GeV} \quad \text{[UMSSM]} \tag{23}$$

Table II shows the lightest neutralino mass ($M_{\chi_1^0}$) and its singlino composition ($|N_{15}|^2$) as well as other relevant parameter values at the bounds. The results in the table are only for positive gaugino masses, but negative gaugino masses do not change these ranges significantly[6].

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[5] We exclude very small M_2 or μ to avoid very light non-singlino states. These are also excluded by the chargino mass constraints.

[6] With $M_2 = -50 \sim -500$ GeV, the χ_1^0 mass ranges are $M_{\chi_1^0} = 39 \sim 254$ GeV (MSSM), $M_{\chi_1^0} = 0 \sim 254$

Figure 1 shows the $M_{\chi_1^0}$ dependence on $\tan\beta$ and $|N_{15}|^2$ (and also $|N_{16}|^2$ for the UMSSM). The dashed lines are the MSSM bounds which neglect the rather weak $\tan\beta$ dependence and use the values of Eq. (20). The mass bound dependence on $\tan\beta$ is quite sensitive in some models. For example, at $\tan\beta \approx 1$, the NMSSM and the UMSSM have massless χ_1^0 while the nMSSM has an upper $M_{\chi_1^0}$ bound; the MSSM violates the LEP2 m_h constraint at this $\tan\beta$. Singlino dominance is typical in the χ_1^0 of the extended MSSM models. Especially, when the $M_{\chi_1^0}$ is much smaller than the MSSM lower limit ($M_{\chi_1^0} \sim 50$ GeV), the singlino is always the dominant component. We will further discuss the allowed mass ranges along with additional indirect constraints in Section V.

IV. INDIRECT CONSTRAINTS ON THE LIGHTEST NEUTRALINOS

A. Cold dark matter relic density and muon anomalous magnetic moment

The cold dark matter density is tightly constrained by the WMAP (CMB) and SDSS (Large Scale Structure) data [18] to be (with 1σ uncertainty)

$$\Omega_{\mathrm{CDM}}h^2 = 0.12 \pm 0.01 \qquad (\mathrm{WMAP} + \mathrm{SDSS}) \tag{24}$$

where $h = 0.72 \pm 0.08$ is the present day Hubble constant H_0 in units of 100 km s^{-1} Mpc^{-1} [19]. We assume that χ_1^0 is the sole dark matter and impose the relic density constraint[7] of Eq. (24).

Another important experimental result that can constrain new physics models is the BNL E821 measurement of the muon anomalous magnetic moment $a_\mu \equiv (g-2)_\mu/2$ [11]. The deviation from the SM is

$$\Delta a_\mu \equiv a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) = (23.9 \pm 10.0) \times 10^{-10} \tag{25}$$

when the SM prediction is based on the hadronic contribution from e^+e^- data. However, the 2.4σ deviation is reduced to 0.9σ if indirect hadronic τ decay data are used instead for the SM prediction.

The dominant Supersymmetric contributions to $(g-2)_\mu$ come from the chargino-sneutrino and the neutralino-smuon loops. The Δa_μ result practically constrains the sign of M_2 to be positive in our sign convention of $\mu_{\mathrm{eff}} > 0$. The predicted $(g-2)_\mu$ value sensitively depends on $\tan\beta$ and the scalar muon mass. The muon trilinear scalar coupling dependence is neglected in our calculation of a_μ.

Typically, a large $\tan\beta$ value is favored to explain the sizable 2.4σ deviation, but with a rather small scalar muon mass, quite small $\tan\beta$ values are also acceptable. Figure 2(a) shows how the acceptable parameter ranges from Figure 1(a) change due to the Δa_μ constraint for two choices of scalar muon mass, $m_L = m_E = 100$ GeV (dots) and 500 GeV (squares).

GeV (NMSSM), $M_{\chi_1^0} = 0.4 \sim 96$ GeV (nMSSM), $M_{\chi_1^0} = 39 \sim 254$ GeV (UMSSM).

[7] It was emphasized in [20] that predicted values of $\Omega_{\mathrm{CDM}}h^2$ lower than the observed range in (24) may be allowed if there are nonthermal production mechanisms.
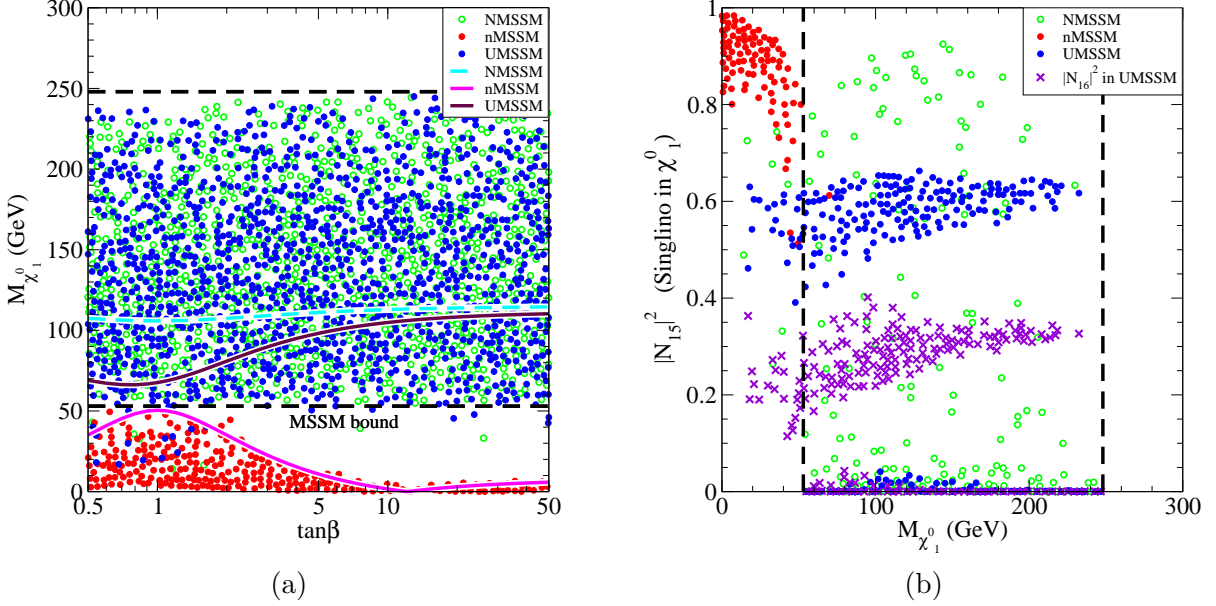
FIG. 1: Scatter plots of (a) the χ_1^0 mass ($M_{\chi_1^0}$) versus $\tan\beta$ and (b) the χ_1^0 singlino composition ($|N_{15}|^2$) versus $M_{\chi_1^0}$ for various models. The crosses in (b) represent the Z'-ino composition ($|N_{16}|^2$) for the UMSSM. The direct constraints of Section III are imposed. The solid curves represent a fixed set of inputs $M_2 = 250$ GeV, $\mu = 250$ GeV, $s = 500$ GeV and $\kappa = 0.5$ (for the NMSSM) without the direct constraints. The upper (lower) UMSSM singlino band in (b) corresponds approximately to moderate (large) value of s, with the lightest neutralino being MSSM-like for large s.

B. Z' boson mass

In $U(1)'$-extended MSSM models, the mass of the new gauge boson Z' is also an important constraint. The Tevatron Run2 dilepton data places a lower bound on the Z' mass of $500 \sim 800$ GeV, with the exact bound depending on the model [21]. In the UMSSM, the Z' mass is given by

$$M_{Z'}^2 = g_{Z'}^2 \left(Q_{H_1}'^2 v_1^2 + Q_{H_2}'^2 v_2^2 + Q_S'^2 s^2 \right). \tag{26}$$

Accordingly, the value of s should be at the few TeV level to satisfy the experimental $M_{Z'}$ bound; this high value of s, with $\mu_{\text{eff}} \sim \mathcal{O}(\text{EW})$, requires very small h_s which implies a fine-tuning.

The bound on s can be significantly reduced if there are additional contributions to $M_{Z'}$. In the S-model, for example, the Z' mass gets additional contributions from 3 more Higgs singlet VEVs $s_{1,2,3}$, which practically removes any lower bound on s if the s_i's are at the TeV-scale.

$$M_{Z'}^2 = g_{Z'}^2 \left(Q_{H_1}'^2 v_1^2 + Q_{H_2}'^2 v_2^2 + Q_S'^2 s^2 + \sum_{i=1}^3 Q_{S_i}'^2 s_i^2 \right) \tag{27}$$

These multiple singlets help to keep $\mu_{\text{eff}} = h_s \frac{s}{\sqrt{2}}$ at the EW scale (resolving the μ-problem) without fine-tuning even for a multi-TeV scale Z'. It is also possible that Z' could be leptophobic, in which case the Tevatron bound on $M_{Z'}$ is greatly reduced. Then s is not severely bounded even if there are no additional contributions to $M_{Z'}$.
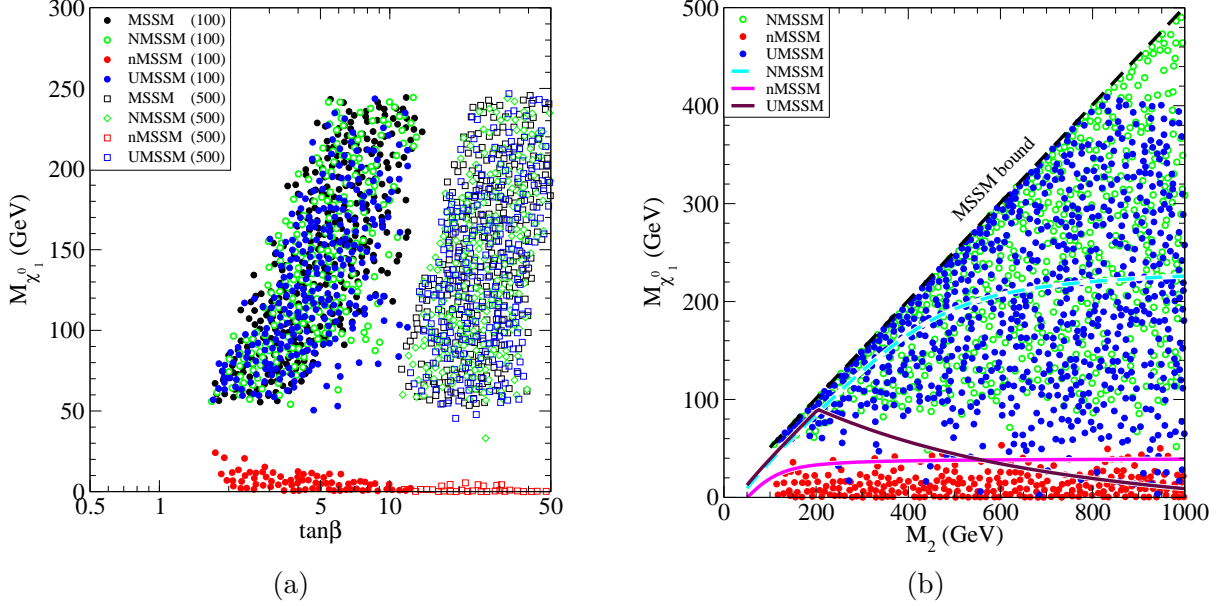
FIG. 2: (a) Similar plot as Figure 1(a) that satisfy the 2.4σ allowed $(g-2)_\mu$ measurement with $m_L = m_E = 100$ GeV (dots) and 500 (GeV) (squares). (b) $M_{\chi_1^0}$ versus M_2 with extended ranges of M_2 and μ up to 1000 GeV. The solid curves are for $\mu = 250$ GeV, $s = 500$ GeV, $\tan\beta = 2$ and $\kappa = 0.5$.

V. DISCUSSION OF THE LIGHTEST NEUTRALINO

A. In the MSSM

The lightest neutralino mass bound in the MSSM for various conditions has been well studied [22]. In our parameter range, the upper bound is 248 GeV, which is associated with the assumed upper bound on M_1. The χ_1^0 mass bound increases if the M_1 bound increases (Figure 2(b)). The lower bound on $M_{\chi_1^0}$ is determined by the light chargino mass bound of $M_{\chi_1^\pm} > 104$ GeV; the bound is $M_{\chi_1^0} > 53$ GeV (roughly half of the chargino mass bound), for which the $\Gamma_{Z\to\chi_1^0\chi_1^0}$ constraint is irrelevant. In this $M_{\chi_1^0}$ range the $\chi_1^0\chi_1^0$ annihilation cross section is of suitable size for the general MSSM to provide an acceptable relic density through multiple annihilation channels [2]. The $(g-2)_\mu$ deviation can be simultaneously explained without difficulty [23]. The LEP2 SM-like Higgs mass bound of $m_h > 114$ GeV excludes $\tan\beta \sim 1$ in the MSSM, though it does not apply to other models with a Higgs singlet.

B. In the NMSSM

Systematic studies of the NMSSM neutralinos can be found in Ref. [24]. The upper bound on the χ_1^0 mass is 248 GeV, as in the MSSM. At this point the singlino component is absent and κ is rather large, 0.65 for $\kappa > 0$ (-0.20 for $\kappa < 0$). With a large enough κs value the singlino component can be very heavy and decoupled, leaving the rest of the matrix similar to the MSSM.
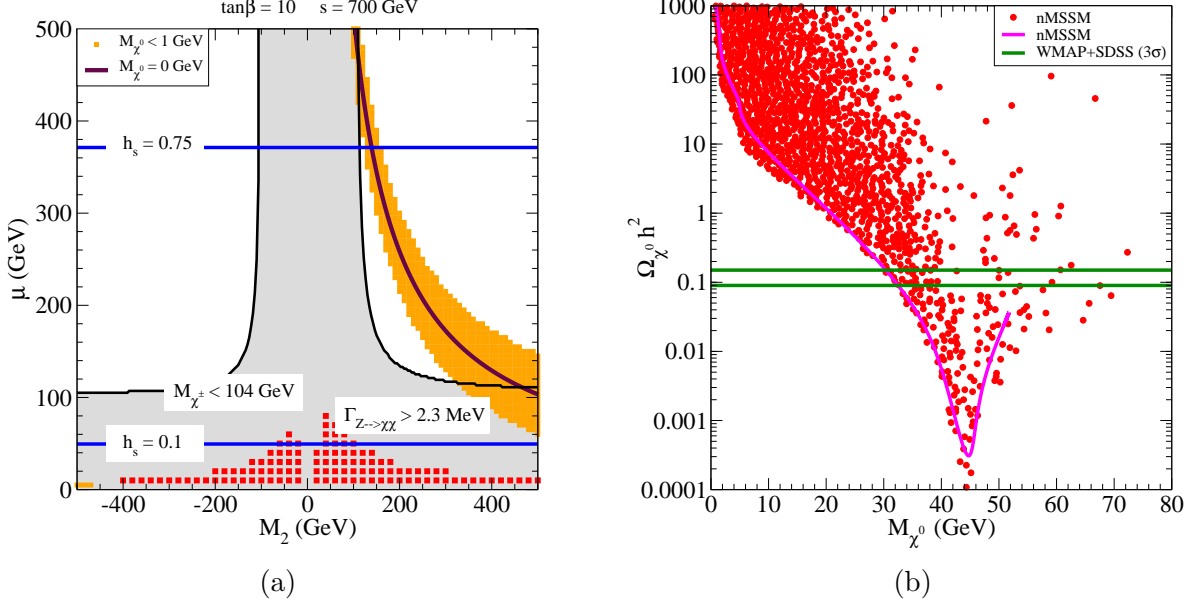
FIG. 3: (a) Illustration of the massless neutralinos (dark curve) and nearby light neutralinos ($M_{\chi_1^0} < 1$ GeV) (orange region) in the M_2-μ plane along with constraints from Γ_Z, $M_{\chi_1^\pm}$ and h_s in the nMSSM or the S-model in the decoupling limit. Here $\tan\beta = 10$ and $s = 700$ GeV are assumed. The gap in the Γ_Z exclusion region is due to the emergence of a very light gaugino for $M_{1,2} \sim 0$. (b) The relic density for the nMSSM or the S-model with Z-pole annihilation. The solid curve is for a fixed set of values of $\mu = 200$ GeV, $s = 400$ GeV and $\tan\beta = 1.5$ with M_2 varying over $0 \sim 500$ GeV.

With positive κ, the lower $M_{\chi_1^0}$ bound is 16 GeV. With negative κ, a massless χ_1^0 occurs at $\tan\beta = 1$ with a singlino composition of $|N_{15}|^2$ of 0.78. Since the massless χ_1^0 state occurs as the result of mass matrix mixing, loop effects such as threshold corrections on the gaugino masses do not prevent the appearance of the massless state. Its mixing with the right-handed neutrinos may result in interesting neutrino physics [25].

This model is disfavored by the non-observation of cosmological domain walls predicted by the discrete symmetries of the model [8]. However, there is an approach to interpret the domain wall as the dark energy [26]. The domain wall network has to be strongly frustrated (nearly static) to satisfy the CMB isotropy constraint. In that case the equation of state of dark energy is expected to be close to $w = -2/3$, which is disfavored by SNIa data and also by joint analysis with other cosmological data [27] but is not excluded.

C. In the nMSSM or the S-model

In the nMSSM (also in the S-model in the decoupling limit), a vanishing lower χ_1^0 mass bound occurs with dominant singlino composition $|N_{15}|^2 \sim 1$, where $\Gamma_{Z \to \chi_1^0 \chi_1^0}$ of Eq. (19) is negligible. As discussed after Eq. (11), the condition for a massless neutralino can be easily satisfied regardless of the $\tan\beta$ and s choices. However, the massless condition for small $\tan\beta$ violates the $M_{\chi_1^\pm} > 104$ GeV constraint. This is why the lower bound is not exactly zero for small $\tan\beta$ in Table II, but can nevertheless be very small for large s. Large s is also
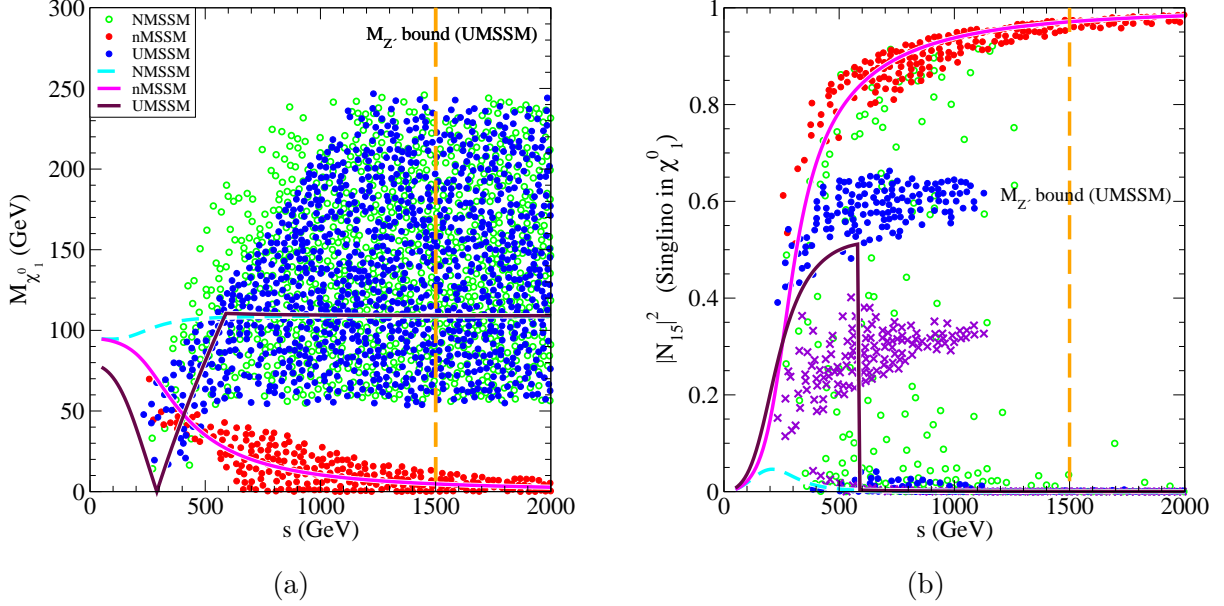
12

FIG. 4: Scatter plots of (a) $M_{\chi_1^0}$ and (b) $|N_{15}|^2$ versus s. The solid curves are for $M_2 = 250$ GeV, $\mu = 250$ GeV, $\tan\beta = 2$ and $\kappa = 0.5$. The $M_{Z'}$ bound $s > 1500$ GeV is approximate and can be evaded, as discussed in Section IV B. The crosses in (b) are the Z'-ino composition ($|N_{16}|^2$) in the UMSSM.

needed for a light χ_1^0 to ensure singlino domination. Figure 3(a) illustrates how the massless or very light neutralinos of $M_{\chi_1^0} < 1$ GeV can appear without violating direct constraints.

The $M_{\chi_1^0}$ upper bound is 83 GeV at $\tan\beta = 1$, which is considerably lower than those of other models[8]; the bound is due to the upper limit imposed on h_s, and the bound significantly increases without this constraint. Figure 4(a) shows that $M_{\chi_1^0}$ decreases with s, as we expect from Eq. (10). As seen in Figure 4(b), the singlino component in χ_1^0 increases with increasing s and becomes dominant for sufficiently large s. While $M_{\chi_1^0}$ increases with decreasing s, s cannot be too small because of the $h_s \leq 0.75$ requirement. The singlino composition at the maximum $M_{\chi_1^0}$ is still dominant with $|N_{15}|^2$ of 0.7. When there are other sizable components in χ_1^0, the $\Gamma_{Z \to \chi_1^0 \chi_1^0}$ constraint is not easy to escape unless $\tan\beta \simeq 1$ where the Z-χ_1^0-χ_1^0 coupling (Eq. (19)) vanishes. At $\tan\beta = 1$ (or $v_1 = v_2$), the contributions of the two doublet Higgsinos to χ_1^0 are identical (up to sign) as Eq. (5) suggests.

Because of the small χ_1^0 mass in this model, most annihilation channels for the MSSM χ_1^0 relic density calculation become irrelevant, and the Z pole is the dominant channel [16, 28]. Figure 3(b) shows the neutralino relic density in this model through the Z pole annihilation. The direct constraints of Section III are applied. To reproduce the acceptable relic density with only the Z pole annihilation contribution, the lower $M_{\chi_1^0}$ bound[9] is $M_{\chi_1^0} \gtrsim 30$ GeV, while the upper bound remains the same as discussed above. This is the most severe lower bound on $M_{\chi_1^0}$ in the model. However, the Higgs masses are not bounded by the LEP2 data because of possible mixing between Higgs doublets and singlets, and very light Higgses may

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[8] Including values with $M_i < 0$, the $M_{\chi_1^0}$ upper bound can increase, but only up to 100 GeV.

[9] The bound can be lowered a bit for a weaker $\Gamma_{Z \to \chi_1^0 \chi_1^0}$ constraint.

provide sufficiently large annihilation so that even lighter neutralinos are allowed by the relic density constraint. For an explicit calculation of such a light neutralino relic density through a light pseudoscalar state, see Ref. [29]. We also refer to Ref. [30] for interesting physics associated with a light pseudoscalar Higgs boson.

As can be seen in Figure 1(a), the lightest neutralinos that are massive enough for the annihilation through the Z pole ($M_{\chi_1^0} \gtrsim 30$ GeV) are allowed only for small $\tan\beta$. This raises concern since the Supersymmetric contribution to $(g-2)_\mu$ is proportional to $\tan\beta$, and a large value of $\tan\beta$ is favored to explain the considerable 2.4σ deviation of the experimental value from the SM prediction. Nonetheless, a common solution was found to exist that can explain both the acceptable relic density through the Z pole and the deviation of $(g-2)_\mu$ in this model, as illustrated in Figure 5(a) [31].

D.   In the UMSSM

In the UMSSM, the upper bound on the lightest neutralino mass is typically given by the maximal value of M_1 (with $s \gg M_1$), where χ_1^0 is essentially the MSSM-like neutralino with almost no singlino composition. The χ_1^0 mass bound does not increase above 420 GeV when the $M_{1,2}$ ranges are extended, unlike the MSSM case (Figure 2(b)). This is because of our restriction $s \leq 2000$ GeV: for the extended $M_{1,2}$ range one is in a regime similar to Eq. (13), with $M_{\chi_1^0}$ given by the smaller of the eigenvalues in Eq. (14). As the solid curves of Figure 4 illustrate, both the mass and the singlino composition of the lightest neutralino increase with s until s reaches a certain value where that neutralino is no longer the lightest and the singlino composition of χ_1^0 plunges to zero.

A vanishing lower bound on $M_{\chi_1^0}$ occurs at $\tan\beta = 1$ and $s = 365$ GeV with a singlino composition of $|N_{15}|^2 \simeq 0.6$ and a Z'-ino composition of $|N_{16}|^2 \simeq 0.2$. Though the singlino component does not saturate χ_1^0 here, the next-to-dominant component, Z'-ino, also does not contribute to the Z decay width, and such a light neutralino survives the $\Gamma_{Z \to \chi_1^0 \chi_1^0}$ constraint.

The Tevatron lower bound on $M_{Z'}$ is too large for the Z' to make a significant contribution to $(g-2)_\mu$ through a Z'-loop, though the μ-χ_i^0-$\tilde{\mu}_j$ contribution is modified by the Z'-ino component in χ_i^0 [31]. Figure 2(a) shows the parameter space constrained by $(g-2)_\mu$ for two values of the smuon mass.

A value of $s \sim \mathcal{O}(\text{EW})$ can be rather problematic, as discussed in Section IV B, unless either there are additional contributions to $M_{Z'}$ (S-model) or the leptonic coupling is small (leptophobic Z' model). If we take a TeV scale lower bound on s (for example, in the $s \gtrsim 1.5$ TeV range in Figure 4) to satisfy the Tevatron $M_{Z'}$ bound of $500 \sim 800$ GeV, the singlino and Z'-ino components are negligible and the lightest neutralino mass becomes similar to that of the MSSM.

VI.   SUMMARY AND CONCLUSION

Although Supersymmetry at the TeV scale is well-motivated, the MSSM is just one of the possible realizations. In fact, the theoretical μ-problem suggests that the MSSM is incomplete. The solution to the μ-problem suggests that an appropriate direction to extend the MSSM is to have an extra Higgs singlet whose VEV gives the effective μ-term of the EW scale. Extensions of the MSSM have extra neutralinos, and the composition of the lightest neutralino involves extra components beyond those of the MSSM. Because of this, both the
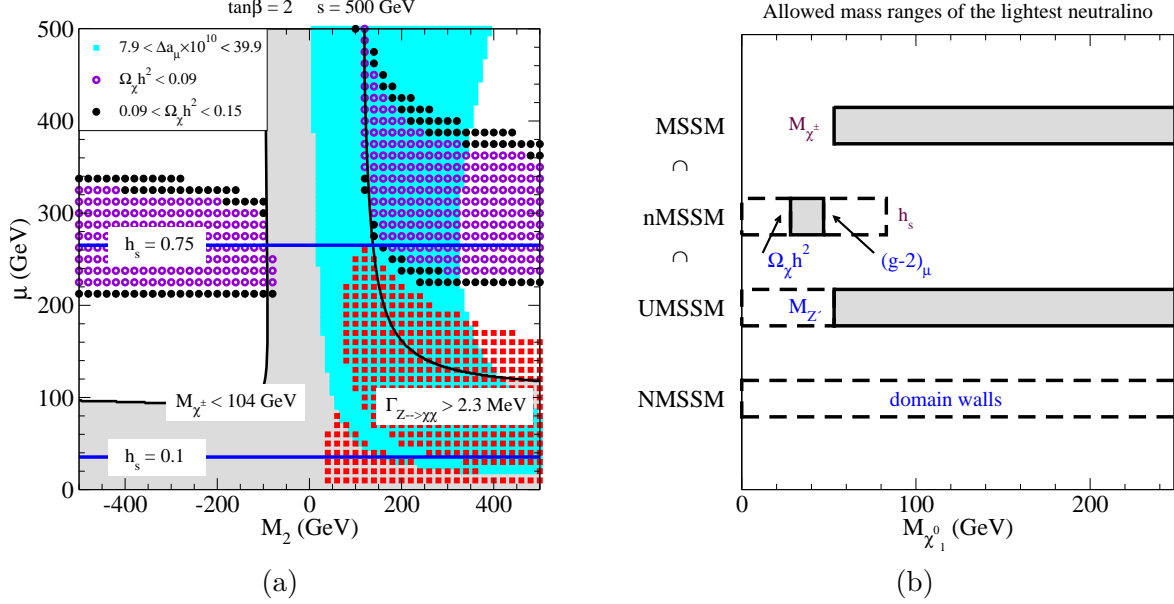
FIG. 5: (a) An illustration of a common solution of the $\Omega_{\chi_1^0} h^2$ (dark dots, 3σ of WMAP+SDSS) and the $(g-2)_\mu$ (cyan region, 95% C.L. of E821) in the M_2-μ plane along with other constraints in the nMSSM or the S-model. The Z-pole annihilation and $m_L = m_E = 100$ GeV are used. The gap in the Γ_Z exclusion region is where $|N_{13}|^2 \approx |N_{14}|^2$. (b) The allowed mass range of χ_1^0 after applying direct constraints (model parameters, gaugino mass unification $M_{1'} = M_1 \simeq 0.5 M_2$, $M_{\chi_1^\pm}$, $\Delta\Gamma_Z$) and indirect constraints ($(g-2)_\mu$, $\Omega_{\chi_1^0} h^2$, $M_{Z'}$, domain wall). The $M_{\chi_1^0}$ bounds in the nMSSM are intended to be illustrative and are not necessarily quantitatively precise.

mass and couplings of the lightest neutralino are modified from the MSSM. The lightest neutralino (χ_1^0) is interesting both in particle physics (as the LSP) and cosmology (as the CDM), and it is therefore important to study and compare properties of the χ_1^0 in extended MSSM models.

We explored χ_1^0 properties in various extended MSSM models. We examined constraints from the experimental bound on the $Z \to \chi_1^0 \chi_1^0$ contribution to Γ_Z, the lower bound on $M_{\chi_1^\pm}$, and constraints from perturbativity and naturalness of the SH_1H_2 and S^3 coupling strengths. We also considered constraints from the relic density of the CDM ($\Omega_{\chi_1^0} h^2$), the experimental deviation of $(g-2)$ of the muon from the SM expectation, and the Tevatron lower bound on $M_{Z'}$. Distinguishing properties of the lightest neutralino arise in extended MSSM models, such as distinct χ_1^0 mass ranges (Eq. (20) - (23)), frequent singlino dominance (Figure 1(b)), importance of the Z pole annihilation channel in the relic density calculation (Figure 3(b)) and different $\tan\beta$ dependences of the upper and lower mass bounds (Figure 1(a)). Some of the extended MSSM models can be considered as limits of the other models. For example, as far as the neutralino sector is concerned, we can consider that MSSM $\subset$ nMSSM $\subset$ UMSSM $\subset$ S-model.

Approximate lightest neutralino mass ranges in the models considered are illustrated in Figure 5(b). The dashed regions are disfavored by the indirect constraints. After the $M_{Z'}$ lower bound is imposed, the UMSSM bound becomes similar to the MSSM. In the case of the nMSSM there is a tension between the $(g-2)_\mu$ constraint which favors small $M_{\chi_1^0}$ (or large $\tan\beta$) and the relic density constraint which favors large $M_{\chi_1^0}$ (or small $\tan\beta$).

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The properties of the CDM particle, even if it is the lightest neutralino, may be quite different from the MSSM prediction. For instance, it could be extremely light and/or dominated by the singlino, which does not directly couple to SM particles except Higgs doublets. Similar distinctions of models may occur in the Higgs sector.

Even if a low energy Supersymmetry is correct, its realization may depend on the model. The measurement of the mass of the lightest neutralino and the determination of its couplings will be particularly useful in testing the MSSM and its extensions at colliders.

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